



NASA Press Release – “The Chandra X-ray Observatory image to the left shows the central region of the supernova remnant Cassiopeia A. This interstellar cloud 14 light years across, is all that remains of a massive star that exploded 330 years ago. A careful analysis of the X-ray data has revealed that the dense neutron star left behind by the supernova has a thin carbon atmosphere as shown in the figure to the right. The neutron star is only 14 miles (23 kilometers) in diameter, and is as dense as an atomic nucleus (100 trillion gm/cc). The atmosphere is only about four inches (10 cm) thick, has a density similar to diamond (3.5 gm/cc), and a temperature of nearly 2 million Kelvin. The surface gravity on the neutron star is 100 billion times stronger than on Earth, which causes the atmosphere to be incredibly thin even with such a high temperature.”

How much carbon is there?

Problem 1 – What are the facts that we know about the atmosphere from the news announcement, and what combination of facts will help us estimate the atmosphere’s mass?

Problem 2 – If the volume of a thin spherical shell is $V = 4 \pi R^2 h$ where R is the radius of the sphere and h is the thickness of the shell, what other formula do you need to calculate the atmosphere’s mass?

Problem 3 – What is your estimate for the mass of the carbon atmosphere in A) kilograms? B) metric tons? C) Earth Atmosphere masses (A_e) where $1 A_e = 5.1 \times 10^{18}$ kg? (Provide answers to two significant figures)

Problem 1 – What are the facts that we know about the atmosphere from the news announcement, and what combination of facts will help us estimate the atmosphere's mass?

Answer: The facts are as follows, with the facts that help estimate the atmosphere's mass indicated in bold face:

- 1... The interstellar cloud is 14 light years across,
- 2... The supernova exploded 330 years ago.
- 3... **The neutron star is only 14 miles (23 kilometers) in diameter.**
- 4... The neutron star is as dense as an atomic nucleus (100 trillion gm/cc).
- 5... **The atmosphere is only about four inches (10 cm) thick,**
- 6... **The atmosphere has a density similar to diamond (3.5 gm/cc),**
- 7... The atmosphere has a temperature of nearly 2 million Kelvin.
- 8... The surface gravity on the neutron star is 100 billion times stronger than on Earth.

Problem 2 – If the volume of a thin spherical shell is $V = 4 \pi R^2 h$ where R is the radius of the sphere and h is the thickness of the shell, what other formula do you need to calculate the atmosphere's mass?

Answer: The formula gives the volume occupied by the atmosphere, so you need a relationship that relates volume to mass: **Mass = Density x Volume.**

Problem 3 – What is your estimate for the mass of the carbon atmosphere in A) kilograms? B) metric tons? C) Earth Atmosphere masses (Ae) where $1 \text{ Ae} = 5.1 \times 10^{18} \text{ kg}$? (Provide answers to two significant figures)

Answer: Convert all measures to centimeters, so the neutron star diameter is 23 kilometers x (100,000 cm/1 km) = $2.3 \times 10^6 \text{ cm}$ and its radius is $1.1 \times 10^6 \text{ cm}$. The volume of the atmosphere is

$$\begin{aligned} V(\text{shell}) &= 4 \times 3.14 \times (1.1 \times 10^6 \text{ cm})^2 (10 \text{ cm}) \\ &= 1.5 \times 10^{14} \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{A) Mass} &= (3.5 \text{ grams/cm}^3) \times 1.5 \times 10^{14} \text{ cm}^3 \\ &= 5.3 \times 10^{14} \text{ grams} \\ &= 5.3 \times 10^{14} \text{ grams} \times (1 \text{ kg}/1000 \text{ grams}) = \mathbf{5.3 \times 10^{11} \text{ kilograms}} \end{aligned}$$

$$\text{B) } 5.3 \times 10^{11} \text{ kilograms} \times (1 \text{ tons}/1000 \text{ kilograms}) = \mathbf{5.3 \times 10^8 \text{ tons (also 0.53 gigatons)}}$$

$$\text{C) } 5.3 \times 10^{11} \text{ kilograms} \times (1 \text{ Ae}/5.1 \times 10^{18} \text{ kilograms}) = \mathbf{1.0 \times 10^{-7} \text{ Ae}}$$